

Spin-dependent energy distribution of B-hadrons from polarized top decays considering the azimuthal correlation rate

S. M. Moosavi Nejad^{a,b*}

^(a)*Faculty of Physics, Yazd University, P.O. Box 89195-741, Yazd, Iran and*

^(b)*School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM), P.O.Box 19395-5531, Tehran, Iran*

(Dated: July 1, 2015)

In our previous work, we studied the polar distribution of the scaled energy of bottom-flavored hadrons from polarized top quark decays $t(\uparrow) \rightarrow W^+ + b(\rightarrow X_b)$, using two different helicity coordinate systems. Basically, the energy distributions are governed by the unpolarized, polar and azimuthal rate functions which are related to the density matrix elements of the decay $t(\uparrow) \rightarrow W^+ + b$. Here we present, for the first time, the analytical expressions for the $\mathcal{O}(\alpha_s)$ radiative corrections to the differential azimuthal decay rates of the partonic process $t(\uparrow) \rightarrow b + W^+ (+g)$ in two helicity systems, which are needed to study the azimuthal distribution of the energy spectrum of the B-hadron produced in polarized top quark decays. Our predictions of the hadron energy distributions enable us to deepen our knowledge of the hadronization process and to determine the polarization states of top quarks.

PACS numbers: 14.65.Ha, 13.88.+e, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

In the Standard Model (SM), the top quark has a short lifetime ($\approx 0.5 \times 10^{-24} s$ [1]) so decays rapidly and this short time does not allow the top quark to form the QCD bound states, phrased in a different language, its short lifetime implies that it decays before hadronization takes place. If it was not for the confinement of color, the top quark could be considered as a free particle and this property allows it to behave like a real particle and one can safely describe its decay in perturbative theory. In fact, at the top mass scale the strong coupling constant is small, $\alpha_s(m_t) \approx 0.107$, so that all QCD effects involving the top quark are well behaved in the perturbative sense. Due to the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element $V_{tb} = 0.999$ [2], the decay width of the top quark is almost exclusively dominated by the two-body channel at the lowest order where a W-gauge boson and a bottom quark are contributed. As it is well known, bottom quarks produced hadronize before they decay ($b \rightarrow X_b$), therefore each X_b -jet contains a bottom-flavored hadron which most of the times is a B-meson. The bottom hadronization is indeed one of the largest sources of uncertainty in the measurement of the mass of top quark at the CERN Large Hadron Collider (LHC) [3] and the Tevatron [4], as it contributes to the Monte Carlo systematics. The LHC is a superlative top factory, which allows us to carry out precision tests of the SM and, specifically, a precise measurement of the top quark properties such as its mass m_t , total decay width Γ_t and branching fractions. At the LHC, of particular interest is the distribution in the energy of meson produced in the top quark rest frame, so that this energy distribution

provides direct access to the bottom fragmentation functions (FFs). In [5], we studied both the B-meson energy distribution produced from unpolarized top decay and we studied the angular distribution of the W-boson decay products in the decay chain $t \rightarrow bW^+ \rightarrow Bl^+\nu_l + X$. Since the top quark decays rapidly so that its life time scale is much shorter than the typical time needed for the QCD interactions to randomize its spin, therefore its full polarization content is preserved when it decays and passes on to its decay products. Hence, the polarization of the top quark will reveal itself in the angular decay distribution and can be studied through the angular correlations between the direction of the top quark spin and the momenta of the decay products, W^+ -boson and b -quark. In [6], we studied the $\mathcal{O}(\alpha_s)$ angular distribution of the scaled energy of the B-hadrons, by calculating the polar angular correlation in the rest frame decay of a polarized top quark into a stable W^+ -boson and B/D-hadrons. We analysed this angular correlation in a helicity coordinate system (system 1) where the event plane, including the top and its decay products, is defined in the (x, z) plane with the z-axes along the bottom quark momentum. In this system the top polarization vector was evaluated with respect to the direction of the bottom quark momentum. Basically, to define the planes we need to measure the momentum directions of the \vec{p}_b and \vec{p}_W and the polarization direction of the top quark, where the evaluation of the momentum direction of \vec{p}_b requires the use of a jet finding algorithm, whereas the top spin direction must be obtained from the theoretical input. For example, in e^+e^- interactions the polarization degree of the top can be tuned with the help of polarized beams [7], so that a polarized linear e^+e^- collider may be considered as a copious source of close to zero and close to 100% polarized tops.

In [8], we analysed the $\mathcal{O}(\alpha_s)$ polar distribution of the B-hadron energy in a different helicity coordinate sys-

*Electronic address: mmoosavi@yazduni.ac.ir

tem (system 2) where, as in [6] the event plane is the (x, z) plane but with the z -axis along the W^+ -boson, so the polarization direction of the top quark is evaluated with respect to the W^+ momentum vector. This election makes the calculation so complicated.

The azimuthal correlations between the event plane and the intersecting ones to this plane evaluated in two helicity systems belong to a class of polarization observables involving the top quark in which the leading-order (LO) contribution gives a zero result in the SM, so the non-zero contributions can either arise from higher order SM radiative corrections or non-SM effects [9]. Since highly polarized top quarks will become available at hadron colliders through single top production processes, which occur at the 33% level of the $t\bar{t}$ pair production rate [10], it will then be possible to experimentally measure the azimuthal correlation between the (\vec{p}_W, \vec{p}_b) and (\vec{P}_t, \vec{p}_b) planes in the helicity system 1 and the (\vec{p}_W, \vec{p}_b) and (\vec{P}_t, \vec{p}_W) planes in the helicity system 2. Here, \vec{P}_t stands for the polarization vector of the top quark and \vec{p}_W and \vec{p}_b stand for the four-momenta of W boson and bottom quark, respectively. To analyse the aforementioned azimuthal correlations in the polarized top rest frame, we study the azimuthal distribution of the scaled energy of B-hadrons at the process $t(\uparrow) \rightarrow W^+ + B + X$ at NLO, by calculating the azimuthal decay distribution of a polarized top quark in the partonic process $t(\uparrow) \rightarrow b + W^+ (+g)$ in two aforementioned coordinate systems. For the nonperturbative part of the process $(b, g \rightarrow B + X)$, from Ref. [11] we apply the realistic $(b, g) \rightarrow B$ FFs obtained through a global fit to e^+e^- data from CERN LEP1 and SLAC SLC. Finally, we shall present and compare our numerical results in both systems. These measurements will be important to deepen our understanding of the nonperturbative aspects of B-hadrons formation and to test the universality and scaling violations of the B-hadron FFs.

This paper is structured as follows. In Sec. II, we introduce the angular rate structure by defining the technical details of our calculations. In Sec. III, our analytic results for the $\mathcal{O}(\alpha_s)$ QCD corrections to the azimuthal distributions of partial decay rates are presented. In Sec. IV, we shall make our predictions of energy distribution of B-hadrons and present our numerical analysis. In Sec. V, our conclusions are summarized.

II. ANGULAR STRUCTURE OF PARTIAL DECAY RATE

In the current-induced $t \rightarrow b$ transition, the dynamics of the process is embodied in the hadron tensor $H^{\mu\nu} \propto \sum_X \langle t | J^{\mu\dagger} | X \rangle \langle X | J^\nu | t \rangle$, where the SM current combination is given by $J_\mu = J_\mu^V - J_\mu^A$. Here, the left-chiral components of the weak current are given by $J_\mu^V \propto \bar{\psi}_b \gamma_\mu \psi_t$ and $J_\mu^A \propto \bar{\psi}_b \gamma_\mu \gamma_5 \psi_t$. In the transition $t \rightarrow W^+ + b(+g)$, the intermediate states are $|X\rangle = |b(p_b)\rangle$ for the Born term and virtual contributions and $|X\rangle = |b + g\rangle$ for

the $\mathcal{O}(\alpha_s)$ real contributions.

The general angular distribution of the differential decay width $d\Gamma/dx$ of a polarized top quark decaying into a jet X_b with bottom quantum numbers and a W^+ boson is expressed by the following form

$$\frac{d\Gamma}{dx_i d\cos\theta_P d\phi_P} = \frac{1}{4\pi} \left\{ \frac{d\Gamma_A}{dx_i} + P \frac{d\Gamma_B}{dx_i} \cos\theta_P + P \frac{d\Gamma_C}{dx_i} \sin\theta_P \cos\phi_P \right\}, \quad (1)$$

where the polar and azimuthal angles θ_P and ϕ_P show the orientation of the plane including the spin of the top quark relative to the event plane (see [8]) and P is the magnitude of the top quark polarization, so $P = 0$ stands for an unpolarized top quark while $P = 1$ corresponds to 100% top quark polarization. In the notation above, $d\Gamma_A/dx$ corresponds to the unpolarized differential decay rate, while $d\Gamma_B/dx$ and $d\Gamma_C/dx$ describe the polar and azimuthal correlation between the polarization of the top quark and its decay products, respectively.

We shall closely follow the notation of [5], where the partonic scaled energy fraction x_i is defined as

$$x_i = \frac{2p_i \cdot p_t}{m_t^2}. \quad (2)$$

As we demonstrated in [5], the finite- m_b corrections are rather small and thus to study the scaled energy distributions of the B-meson, we employ the massless scheme or zero-mass variable-flavor-number (ZM-VFN) scheme [12] in the top quark rest frame, where the zero mass parton approximation is also applied to the bottom quark. The non-zero value of the b -quark mass only enter through the initial condition of the nonperturbative FFs. Nonperturbative FFs are describing the hadronization processes $(b, g) \rightarrow X_b$ and are subject to Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution [13].

By the zero mass approximation, one has $0 \leq x_i \leq 1 - \omega$ where $\omega = m_W^2/m_t^2$. Throughout this manuscript, we apply the normalized partonic energy fraction as

$$x_i = \frac{2E_i}{m_t(1 - \omega)}, \quad (i = b, g) \quad (3)$$

where E_i refers to the energy of outgoing partons (bottom or gluon) and $0 \leq x_i \leq 1$.

In our previous works, the NLO radiative corrections to the unpolarized differential rate $d\Gamma_A/dx_i$ [5] and the polar differential rates $d\Gamma_B/dx_i$ [6, 8] have been studied in two possible helicity systems, extensively. In the present work, we study the radiative corrections to the azimuthal correlation function $d\Gamma_C/dx_i$ in both helicity systems, which have not been done before. Finally, at the hadron level we shall compare our predictions for the energy distribution of B-mesons in two coordinate systems 1 and 2, considering all contributions.

III. ANALYTIC RESULTS FOR AZIMUTHAL DECAY DISTRIBUTIONS

In the rest frame of a top quark decaying into a W^+ boson, a b-quark and a gluon, the final state particles define an event plane. Relative to this plane one can then define the spin direction of the polarized top quark. There are two different choices of possible coordinate systems relative to the event plane where one differentiates between helicity systems according to the orientation of the z -axis (these systems compared in [8]). Four-momenta of the b-quark and the W^+ boson in these two various coordinate systems are defined as

$$\begin{aligned} \text{System 1 : } & \vec{p}_b \parallel z \quad ; (\vec{p}_W)_x \geq 0 \\ \text{System 2 : } & \vec{p}_W \parallel z \quad ; (\vec{p}_b)_x \geq 0 \end{aligned} \quad (4)$$

Indeed, in the system 1 the three-momentum of the b-quark points into the direction of the positive z -axis and in the system 2, the three-momentum of the W^+ boson is defined along this axis.

In the following, we explain the technical details of our calculation for the NLO radiative corrections to the tree-level decay rate of $t(\uparrow) \rightarrow b + W$.

A. Born term results

In the SM, the polarized top decay rate is dominated by the decay process $t(p_t) \rightarrow b(p_b) + W^+(p_W)$ at the Born level. In the rest frame of the top quark, the four-momentum of the top quark is set to $p_t = (m_t; \vec{0})$ and the polarization four-vector of the top quark is set as $s_t = P(0; \sin \theta_P \cos \phi_P, \sin \theta_P \sin \phi_P, \cos \theta_P)$, where P is the top polarization degree ($0 \leq P \leq 1$). Considering the coordinate system 1, where the three-momentum of the b-quark points into the positive z -axis, we set the four-momentum of the b-quark as $p_b = E_b(1; 0, 0, 1)$ and in the system 2, it is $p_b = E_b(1; 0, 0, -1)$ where the three-momentum of the W^+ boson is defined along the positive z -axis. Note that by applying the ZM-VFN scheme we put the b-quark mass to zero throughout this paper. Thus, the Born term helicity structure of partial rates, reads

$$\begin{aligned} \frac{d^2 \Gamma^{(0)}}{d \cos \theta_P d \phi_P} &= \frac{1}{4\pi} \left\{ \Gamma_A^{(0)} \mp P \Gamma_B^{(0)} \cos \theta_P \right. \\ &\quad \left. + P \Gamma_C^{(0)} \sin \theta_P \cos \phi_P \right\}, \end{aligned} \quad (5)$$

where the sign “ $-$ ” stands for the helicity system 1 and the sign “ $+$ ” is for the second one, and

$$\begin{aligned} \Gamma_A^{(0)} &= \frac{m_t \alpha}{16 \omega \sin^2 \theta_W} (1 + 2\omega)(1 - \omega)^2, \\ \Gamma_B^{(0)} &= \frac{m_t \alpha}{16 \omega \sin^2 \theta_W} (1 - 2\omega)(1 - \omega)^2, \\ \Gamma_C^{(0)} &= 0 \end{aligned} \quad (6)$$

where, θ_W is the weak mixing angle and α is the tiny structure constant. These results are in complete agreement with the expressions in [14, 15].

As it is seen, the Born term contribution to Γ_C is zero. We point out that the vanishing of this azimuthal correlation is a consequence of the left-chiral (V-A)(V-A) nature of the current-current interaction in the SM. Another example of a LO zero polarization observable is given in [5]. There, we showed that the decay of a top quark into a polarized transverse-plus W boson and a (massless) bottom quark leads to the contribution zero for the top decay rate into the transverse-plus W boson at the Born term level due to the left-chiral (V-A) coupling structure of the SM. However, if one takes a massive b-quark in the calculation, this contribution is no longer zero but the LO result obtained for Γ_C does not depend on the mass of the bottom quark.

B. QCD NLO contribution to the azimuthal differential decay rate $d\Gamma_c/dx_i$

Generally, the required ingredients for the NLO perturbative calculation are the virtual one-loop contributions and the tree-graph (real emission) contributions. Since, at LO the relevant scalar products are $p_t \cdot s_t = 0$, $p_t \cdot p_b = m_t E_b$ and $p_b \cdot s_t = \mp P E_b \cos \theta_P$ (in both helicity systems) then the virtual one-loop corrections are contributed in the unpolarized rate (Γ_A) and the polar correlation function (Γ_B), which have been studied extensively before [6], while the azimuthal one (Γ_C) does not have any contribution from the virtual corrections.

The QCD NLO contribution results from the square of the real gluon emission graphs. By working in the massless scheme where $m_b = 0$, for the corresponding real amplitude squared one has

$$\begin{aligned} |M^{\text{real}}|^2 &= -\frac{\pi^2 C_F \alpha \alpha_s}{4 \sin^2 \theta_W} (-g^{\mu\nu} + \frac{p_W^\mu \cdot p_W^\nu}{m_W^2}) \times \\ &\quad \left\{ \frac{F_1}{(p_t \cdot p_g)^2} + \frac{F_2}{(p_b \cdot p_g)^2} - \frac{2F_3}{(p_t \cdot p_g)(p_b \cdot p_g)} \right\}, \end{aligned} \quad (7)$$

where $C_F = 4/3$ stands for the color factor, and

$$\begin{aligned} F_1 &= \text{Tr}[\not{p}_b \gamma_\mu (1 - \gamma_5)(m_t + \not{p}_t - \not{p}_g) \gamma^\beta (\not{p}_t + m_t) \times \\ &\quad (1 + \gamma_5 \not{s}_t) \gamma_\beta (m_t + \not{p}_t - \not{p}_g) (1 + \gamma_5) \gamma_\nu], \\ F_2 &= \text{Tr}[\not{p}_b \gamma^\beta (\not{p}_b + \not{p}_g) \gamma_\mu (1 - \gamma_5)(m_t + \not{p}_t) (1 + \gamma_5 \not{s}_t) \times \\ &\quad (1 + \gamma_5) \gamma_\nu (\not{p}_b + \not{p}_g) \gamma_\beta], \\ F_3 &= \text{Tr}[\not{p}_b \gamma_\mu (1 - \gamma_5)(m_t + \not{p}_t - \not{p}_g) \gamma^\beta (m_t + \not{p}_t) \times \\ &\quad (1 + \gamma_5 \not{s}_t) (1 + \gamma_5) \gamma_\nu (\not{p}_b + \not{p}_g) \gamma_\beta]. \end{aligned} \quad (8)$$

In general, to regulate the gluon IR singularities we work in a D -dimensions approach, where the differential decay rate for the real contribution is given by

$$d\Gamma = \frac{\mu_F^{2(4-D)}}{2m_t} |M^{\text{real}}|^2 dR_3(p_t, p_b, p_g, p_W), \quad (9)$$

where, μ is an arbitrary reference mass and the 3-body phase space element dR_3 reads

$$\frac{d^{D-1}\mathbf{p}_b}{2E_b} \frac{d^{D-1}\mathbf{p}_W}{2E_W} \frac{d^{D-1}\mathbf{p}_g}{2E_g} (2\pi)^{3-2D} \delta^D(p_t - \sum_{g,b,W} p_f). \quad (10)$$

Here, $d^{D-1}|\mathbf{p}| = |\vec{p}|^{D-2} d|\vec{p}| d\Omega$ where the angular integral in D-dimensions will have to be written as

$$\frac{d\Omega}{d\phi_P d\cos\theta_P} = -\frac{2\pi^{\frac{D-3}{2}}}{\Gamma(\frac{D-3}{2})} (\sin\theta_P)^{D-4} (\sin\phi_P)^{D-4}. \quad (11)$$

Considering the general form of the angular decay distribution (1), as we showed in [8] the unpolarized differential decay rate $d\Gamma_A/dx$ is independent of the applied helicity system but the polar distribution of decay width $d\Gamma_B/dx$ depends on the various choices of possible coordinate systems, but all final results are free of IR singularities. In the following we will concentrate on the differential azimuthal correlation function $d\Gamma_C/dx$, considering both helicity coordinate systems.

In the system 1, the relevant scalar products are

$$\begin{aligned} p_g \cdot s_t &= -PE_g(\sin\theta_{gb}\sin\theta_P\cos\phi_P + \cos\theta_{gb}\cos\theta_P), \\ p_g \cdot p_b &= E_g E_b(1 - \cos\theta_{gb}), \\ p_b \cdot s_t &= -PE_b\cos\theta_P, \end{aligned} \quad (12)$$

where θ_{gb} is the polar angle between the gluon and the bottom quark momenta in the event plane, so $\cos\theta_{gb} = (m_t^2 - m_W^2 - 2m_t(E_b + E_g) + 2E_b E_g)/(2E_b E_g)$. To calculate the $d\Gamma_C/dx_b$, in (9) we fix the momentum of the b-quark and integrate over the energy of the gluon, which ranges as $m_t S(1 - x_b) \leq E_g \leq m_t S(1 - x_b)/(1 - 2Sx_b)$, where $S = (1 - \omega)/2$. Therefore, one has

$$\begin{aligned} \frac{d\Gamma_{1C}}{dx_b} &= \Gamma_B^{(0)} \frac{\alpha_s C_F}{2(1 - \omega)(1 - 2\omega)} \left\{ 4(\omega - 1)x_b - 2\omega^2 - 5\omega \right. \\ &\quad + 11 - \frac{4(1 + \omega)}{x_b} + \frac{1}{(1 - x_b(1 - \omega))^{\frac{3}{2}}} \left[-16 + \right. \\ &\quad \left. 2\omega + 10\omega^2 + \frac{4(1 + \omega)}{x_b} + 2(1 - \omega)^2(\omega - 5)x_b^2 + \right. \\ &\quad \left. (1 - \omega)^3 x_b^3 + (7\omega^3 - 3\omega^2 - 25\omega + 21)x_b \right] \Big\}, \end{aligned} \quad (13)$$

where x_b is defined in (3). This result can be compared against known results presented in [14] after integrating over $x_b(0 \leq x_b \leq 1)$.

Since the observed mesons in top quark decays can be also produced through a fragmenting real gluon, therefore, to obtain the most accurate energy distribution of the B meson one has to add the contribution of gluon fragmentation to the b-quark one. In [6], it is shown that the gluon contribution can be important at a low energy

of the detected meson so that this contribution decreases the size of decay rate at the threshold energy. $d\Gamma_A/dx_g$ is the same in both coordinate systems and can be found in [5], and the analytical expression for the $d\Gamma_{1B}/dx_g$ in the helicity system 1 is presented in [6] and the $d\Gamma_{2B}/dx_g$ is given in [8], where x_g is defined in (3). In the coordinate system 1, the azimuthal differential width $d\Gamma/dx_g$ reads

$$\begin{aligned} \frac{d\Gamma_{1C}}{dx_g} &= \Gamma_B^{(0)} \frac{\alpha_s C_F}{(1 - \omega)(1 - 2\omega)} \left\{ 4(1 - \omega)x_g - 8 + 6\omega + \right. \\ &\quad \frac{2(1 + \omega)}{x_g} + \frac{1 - 2\omega}{x_g^2} + \frac{1}{4(1 - x_g(1 - \omega))^{\frac{3}{2}}} \left[\right. \\ &\quad -7(1 - \omega)^3 x_g^3 - 2(1 - \omega)^2(5\omega - 19)x_g^2 - \\ &\quad (9\omega^3 + 19\omega^2 - 93\omega + 65)x_g - \frac{4(1 - 2\omega)}{x_g^2} + \\ &\quad \left. \left. 2(2\omega^3 - 13\omega^2 - 5\omega + 20) + \frac{2(6\omega^2 - 13\omega - 1)}{x_g} \right] \right\}. \end{aligned} \quad (14)$$

In the helicity coordinate system 2, the relevant scalar products are

$$\begin{aligned} p_g \cdot s_t &= PE_g(\sin\theta_{gW}\sin\theta_P\cos\phi_P - \cos\theta_{gW}\cos\theta_P), \\ p_b \cdot s_t &= PE_b(\sin\theta_{bW}\sin\theta_P\cos\phi_P - \cos\theta_{bW}\cos\theta_P), \\ p_g \cdot p_W &= m_t(E_W + E_g) - \frac{m_t^2 + m_W^2}{2}, \\ p_b \cdot p_W &= m_t(E_W + E_b) - \frac{m_t^2 + m_W^2}{2}, \\ p_W \cdot s_t &= -P|\vec{p}_W|\cos\theta_P, \end{aligned} \quad (15)$$

and $p_t \cdot s_t = 0$. In the system 2, θ_{bW} is the polar angle between the b-quark momentum and the W boson (z-axis) and θ_{gW} is the angle between the gluon and the W boson, whereas $\cos\theta_{gW} = (m_t^2 + m_W^2 - 2m_t(E_W + E_g) + 2E_W E_g)/(2E_g p_W)$ and $\cos\theta_{bW} = (m_t^2 + m_W^2 - 2m_t(E_b + E_W) + 2E_b E_W)/(2E_b p_W)$ with $p_W = \sqrt{E_W^2 - m_W^2}$.

As before, to calculate the azimuthal differential rate $d\Gamma_C/dx_b$, in (9) we fix the momentum of the b-quark but we integrate over the energy of the W boson, which ranges as $m_t(\omega + [1 - 2Sx_b]^2)/(2(1 - 2Sx_b)) \leq E_g \leq m_t(1 - S)$. Therefore, in the coordinate system 2 the azimuthal differential width $d\Gamma_C/dx_b$ is expressed as

$$\begin{aligned} \frac{d\Gamma_{2C}}{dx_b} &= \Gamma_B^{(0)} \frac{\alpha_s C_F}{2(2 - (1 - \omega)x_b)^2(1 - \omega)(1 - 2\omega)} \times \\ &\quad \left\{ x_b^2(1 - \omega)^2(2\omega^2 - 3\omega - 3) + 4\omega^3 + 22\omega^2 - 16\omega \right. \\ &\quad - 2 + 2x_b(1 - \omega)(-6\omega^2 + 7\omega + 3) - \\ &\quad 2\sqrt{\frac{\omega + (1 - x_b(1 - \omega))^2}{1 + \omega}} \left[x_b^2(1 - \omega)^2(2\omega - 1) + \right. \\ &\quad \left. 2x_b(\omega - 1)(2\omega^2 + 4\omega - 1) + (1 + \omega)(2\omega^2 + 9\omega - 1) \right] \\ &\quad \left. + 8\omega(2 - x_b(1 - \omega))^2 \sqrt{1 - x_b(1 - \omega)} \right\}, \end{aligned} \quad (16)$$

and for the gluon one, we have

$$\begin{aligned} \frac{d\Gamma_{2C}}{dx_g} = & \Gamma_B^{(0)} \frac{\alpha_s C_F}{(1-\omega)(1-2\omega)} \left\{ \frac{1}{2x_g \sqrt{1-x_g(1-\omega)}} \left[\right. \right. \\ & \left. \omega(1-x_g^2) - (1-2\omega^2)(1-x_g)^2 \right] + \\ & \left(\sqrt{\frac{\omega + (1-x_g(1-\omega))^2}{1+\omega}} - \sqrt{1-x_g(1-\omega)} \right) \\ & \left. \times \left[2\omega - 1 - \frac{(1+\omega)^2(1-2\omega)}{x_g^2(1-\omega)^2} + \frac{2(1+2\omega^2)}{x_g(1-\omega)} \right] \right\}. \end{aligned} \quad (17)$$

IV. NUMERICAL ANALYSIS

By determining the differential decay rates in the parton level, in the first step we turn to our numerical predictions of the unpolarized and polarized decay rates by integrating $d\Gamma/dx_b$ over $x_b(0 \leq x_b \leq 1)$, while the strong coupling constant is evolved from $\alpha_s(m_Z) = 0.1184$ to $\alpha_s(m_t) = 0.1070$. By combining our results for the differential azimuthal correlation functions (Eqs. (13) and (16)) with the results obtained for the unpolarized rate [5] and the polar correlation rate in the system 1 [6], one has

$$\begin{aligned} \frac{d\Gamma_1^{\text{NLO}}}{d\phi_{1P} d\cos\theta_{1P}} = & \frac{1}{4\pi} \left\{ \Gamma_A^{(0)}(1 - 0.08542) \right. \\ & - \Gamma_B^{(0)}(1 - 0.1303)P\cos\theta_{1P} \\ & \left. - \Gamma_B^{(0)}(0.0980)P\sin\theta_{1P}\cos\phi_{1P} \right\} \\ = & \frac{\Gamma_A^{\text{NLO}}}{4\pi} \left\{ 1 - 0.3777P\cos\theta_{1P} \right. \\ & \left. - 0.0426P\sin\theta_{1P}\cos\phi_{1P} \right\}, \end{aligned} \quad (18)$$

and by considering the polar correlation one in the system 2 [8], one has

$$\begin{aligned} \frac{d\Gamma_2^{\text{NLO}}}{d\phi_{2P} d\cos\theta_{2P}} = & \frac{1}{4\pi} \left\{ \Gamma_A^{(0)}(1 - 0.08542) \right. \\ & + \Gamma_B^{(0)}(1 - 0.2814)P\cos\theta_{2P} \\ & \left. + \Gamma_B^{(0)}(0.01446)P\sin\theta_{2P}\cos\phi_{2P} \right\} \\ = & \frac{\Gamma_A^{\text{NLO}}}{4\pi} \left\{ 1 + 0.3121\cos\theta_{1P} \right. \\ & \left. + 0.0063P\sin\theta_{1P}\cos\phi_{1P} \right\}, \end{aligned} \quad (19)$$

where $\Gamma_A^{(0)} = 1.4705$ and $\Gamma_B^{(0)} = 0.5841$ if one sets $m_W = 80.399$ GeV, $m_t = 172.98$ GeV and $\sin^2\theta_W = 0.2312$ [16]. As it is seen, the azimuthal correlation generated by the radiative corrections is quite small, especially in the second coordinate system. We can assert that, if top quark decays reveal a violation of the SM (V-A) current structure in the azimuthal correlation function which exceeds

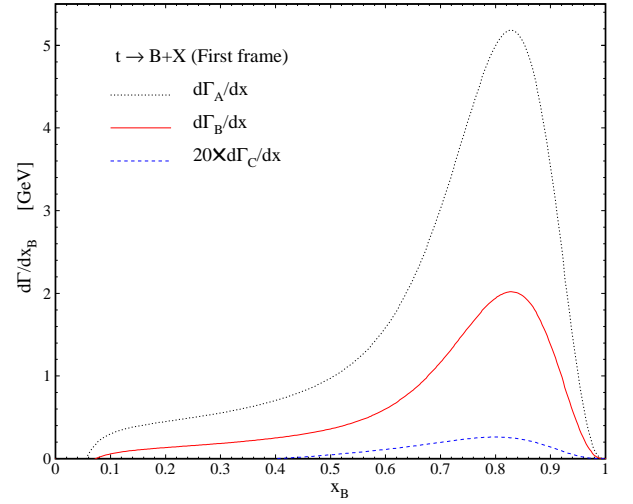


Figure 1: x_B distribution of $d\Gamma^{\text{NLO}}/dx_B$ in the helicity system 1, considering the unpolarized (dotted line), the polar (solid line) and the azimuthal (dashed line) contributions.

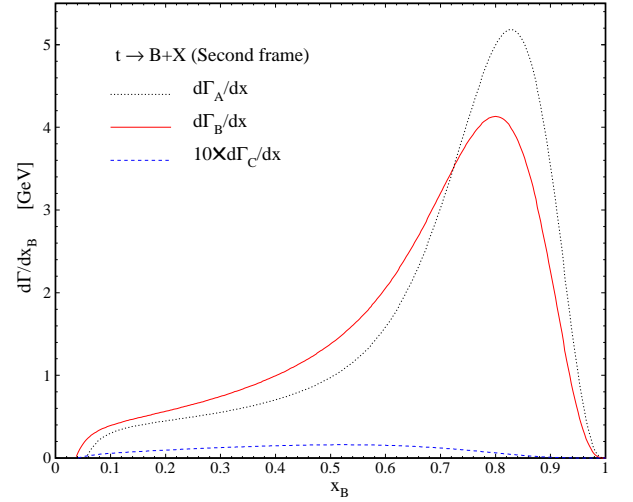


Figure 2: As in Fig 1, but in the helicity system 2.

the 5% level at the system 1 and the 1% level at the system 2, the violation must have a non-SM origin.

In the last step, we present our phenomenological results for the energy spectrum $d\Gamma/dx_B$ of the B meson, where we define the normalized energy fraction of the B meson as $x_B = 2E_B/(m_t(1-\omega))$ (3). According to the factorization theorem of the QCD-improved parton model [17], the energy distribution of B meson can be expressed as the convolution of the parton-level spectrum $d\Gamma/dx_i$ with the nonperturbative fragmentation functions $D_i^B(z, \mu_F)$ as

$$\frac{d\Gamma}{dx_B} = \sum_{i=b,g} \frac{d\Gamma}{dx_i}(\mu_R, \mu_F) \otimes D_i^B\left(\frac{x_B}{x_i}, \mu_F\right). \quad (20)$$

The integral convolution is defined as $(f \otimes g)(x) =$

$\int_x^1 dx f(z)g(x/z)$. In (20), μ_F and μ_R are the factorization and the renormalization scales, respectively, that the scale μ_R is associated with the renormalization of the strong coupling constant and a choice often made consists of setting $\mu_R = \mu_F$. As in our previous works, we adopt the convention $\mu_R = \mu_F = m_t$.

In (20), $D_i(z, \mu_F)$ is the nonperturbative FF describing the transition $(b, g) \rightarrow B$ which is process independent. Several models are proposed to describe the nonperturbative transition from a parton into a hadron state. Here, following Ref. [11] we employ the B meson FF determined at NLO in the ZM-VFN scheme and obtained by fitting the experimental data from the ALEPH and OPAL collaborations at CERN LEP1 and by SLD at SLAC SLC. Authors in [11] have parametrized the z distribution of the $b \rightarrow B$ FF at the initial scale $\mu_0 = m_b$ as $D_b^B(z, \mu_0) = N z^\alpha (1-z)^\beta$ (power model), while the gluon FF is set to zero at the starting scale and is evolved to higher scales using the DGLAP equations [13]. Their results for the fit parameters at the initial scale are $N = 4684.1$, $\alpha = 16.87$ and $\beta = 2.628$ with $\chi^2 = 1.495$. Following Ref. [16], as numerical input values we take $m_W = 80.339$ GeV, $m_b = 4.78$ GeV, $m_B = 5.279$ GeV and the typical QCD scale $\Lambda_{\overline{MS}}^{(5)} = 231$ MeV. Note that, in the ZM-VFN scheme the b-quark mass only enter through the initial condition of the FF.

To study the x_B scaled energy distributions of B mesons, we consider the quantity $d\Gamma(t(\uparrow) \rightarrow B + X)/dx_B$ in the two helicity coordinate systems. In [5, 6], we showed that the $g \rightarrow B$ contribution into the NLO energy spectrum of the B-meson is negative and appreciable only in the low- x_B region and for higher values of x_B the NLO result is practically exhausted by the $b \rightarrow B$ contribution. The contribution of the gluon is calculated to see where it contributes to $d\Gamma/dx_B$ and can not be discriminated in the meson spectrum as an experimental quantity. In the scaled energy of mesons, all contributions including the bottom quark, gluon and light quarks contribute. In Fig. 1, the x_B -spectrum of the B meson produced through the unpolarized top quark decay (dotted line) is shown. The polar (solid line) and the azimuthal

(dashed line) contributions in the helicity system 1 are also studied. As we explained, the azimuthal correlation is prohibited at LO, which explains the smallness of the corresponding result. Note that the threshold occurs at $x_B \geq 2m_B/(m_t(1-\omega)) = 0.078$. In Fig. 2, the same predictions are shown in the helicity system 2.

V. CONCLUSIONS

To study the $\mathcal{O}(\alpha_s)$ spin-dependent energy spectrum of hadrons produced from polarized top quark decays, one needs to know the NLO radiative corrections to the angular differential decay rates of the process $t(\uparrow) \rightarrow b + W^+$. In our previous works, the unpolarized decay rate ($d\Gamma_A/dx_i$) and the polar correlation one ($d\Gamma_B/dx_i$) were calculated at the parton-level in two different helicity coordinate systems. These various helicity systems provide independent probes of the polarized top quark decay dynamics.

Here, by considering two helicity systems we have calculated the $\mathcal{O}(\alpha_s)$ corrections to the differential azimuthal correlation function ($d\Gamma_C/dx_i$) which vanishes at the Born term level. These quantities are required to calculate the x_B distribution $d\Gamma_C/dx_B$ of $t \rightarrow B + X$. Comparing future measurements of the polarized and unpolarized partial widths $d\Gamma/dx_B$ at the LHC with our NLO predictions, one will be able to test the universality and scaling violations of the B meson FFs. These measurements will finally be the primary source of information on the B meson FFs and the azimuthal correlation function $d\Gamma_C/dx_B$ can also constrain the $g \rightarrow B$ and $b \rightarrow B$ FFs even further.

We also found that the $\mathcal{O}(\alpha_s)$ radiative corrections to the azimuthal correlation observable Γ_C are so small, especially in the helicity system 2. Specifically, it is safe to say that, if top quark decays reveal a violation of the SM (V-A) current structure in the azimuthal correlation function which exceeds the 5% level at the system 1, the violation must have a non-SM origin.

-
- [1] K. G. Chetyrkin, R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Rev. D **60** (1999) 114015 [hep-ph/9906273].
 - [2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
 - [3] M. Beneke, I. Efthymiopoulos, M.L. Mangano, J. Womersley *et al.*, in Proceedings of 1999 CERN Workshop on Standard Model Physics (and more) at the LHC, CERN 2000-004, G. Altarelli and M.L. Mangano eds., p. **419**, [hep-ph/0003033].
 - [4] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **96** (2006) 022004; V. M. Abazov *et al.* [D0 Collaboration], Phys. Lett. B **606** (2005) 25.
 - [5] B. A. Kniehl, G. Kramer and S. M. M. Nejad, Nucl. Phys. B **862**, 720 (2012) arXiv:1205.2528 [hep-ph].
 - [6] S. M. M. Nejad, Phys. Rev. D **88** (2013) 9, 094011 [arXiv:1310.5686 [hep-ph]].
 - [7] S. J. Parke and Y. Shadmi, Phys. Lett. B **387** (1996) 199 [hep-ph/9606419].
 - [8] S. M. Moosavi Nejad and M. Balali, Phys. Rev. D **90** (2014) 11, 114017 [arXiv:1409.1389 [hep-ph]].
 - [9] S. Groote, W. S. Huo, A. Kadeer and J. G. Körner, Phys. Rev. D **76** (2007) 014012.
 - [10] G. Mahlon and S. J. Parke, Phys. Rev. D **55**, 7249 (1997).
 - [11] B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Phys. Rev. D **77**, 014011 (2008).

- [12] J. Binnewies, B.A. Kniehl, and G. Kramer, Phys. Rev. D **58**, 034016 (1998);
M. Cacciari and M. Greco, Nucl. Phys. B **421**, 530(1994).
- [13] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972) [Yad. Fiz. **15**, 781 (1972)]; G. Altarelli and G. Parisi, Nucl. Phys. B **126**, 298 (1977); Yu. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977) [Zh. Eksp. Teor. Fiz. **73**, 1216 (1977)].
- [14] M. Fischer, S. Groote, J. G. Körner and M. C. Mauser, Phys. Rev. D **65**, 054036 (2002).
- [15] M. Fischer, S. Groote, J. G. Körner, M. C. Mauser and B. Lampe, Phys. Lett. B **451**, 406 (1999).
- [16] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
- [17] J. C. Collins, Phys. Rev. D **66** (1998) 094002.